Bayesian Tracking and Multi-Core Beamforming for Estimation of Correlated Brain Sources

Petia Georgieva*, Filipe Silva*, Nidhal Bouaynaya†, Lyudmila Mihaylova**

**Institute of Electronics Engineering and Telematics of Aveiro (IEETA), Department of Electronics, Telecommunications and Informatics(DETI), University of Aveiro, 3810-193 Aveiro, Portugal (e-mail: petia@ua.pt; fmsilva@ua.pt)*
† College of Engineering, Rowan University, USA (e-mail: bouaynaya@rowan.edu).
** *Department of Automatic Control and Systems Engineering, University of Sheffield, Mappin Street, Sheffield S1 3JD,UK (e-mail: L.S.Mihaylova@sheffield.ac.uk)*

Keywords: Bayesian tracking, particle filtering, multi-core beamforming, brain sources.

Abstract

The main contribution of this paper is the general framework, termed multi-core Beamformer Particle Filter (multi-core BPF), for solving the ill-posed EEG inverse problem. The method combines a particle filter (statistical approach) for reconstruction of the brain source spatial locations and a multi-core Beamformer (deterministic approach) for estimation of the corresponding dipole waveforms in a recursive way The intuition behind is to benefit from the advantages of both deterministic and statistical inverse problem solvers in order to improve the estimation accuracy without increasing the complexity and the computational cost. Our simulations show that the proposed algorithm can reconstruct reliably the few most active (the dominant) brain sources that have generated the registered EEG measurements. The main advantage of the method is that in contrast to conventional (single-core) Beamforming spatial filters, the proposed Multi-core Beamformer explicitly takes into consideration potential temporal correlation between the dipoles.

1 Introduction

Brain source reconstruction is the process of localization and tracking of the brain electrical activity of the localized sources. The reconstruction of the neuronal activity can be very useful for example during brain surgery of patients, or for Parkinson's disease, where the neural dipole tracking can identify and locate the exact source of the electrical nerve signals and thus improve the deep brain stimulation treatment. The brain source reconstruction techniques are most generally divided into [1] i) the imaging approaches, where the neural activity is described by a dense set of dipoles and ii) the parametric approaches, which employ a small number of equivalent current dipoles. The imaging approaches are by far more researched because they provide a detailed map of the brain neuronal activity. However, it usually takes more than one imaging modality, often using invasive techniques, to obtain a map of the brain neuronal activity. As a consequence, the imaging approaches require invasive as well as computationally involved procedures.

The parametric approaches, on the other hand, are less studied. Their main advantage is that they approximate aggregated event related potentials; and thus represent the neural activity at a macro level. First, deterministic source reconstruction was proposed, based on the principal/independent component analysis or blind source separation techniques [1]. It is only recently, and due to the increase in the available computational power, that statistical parametric methods, such as the Kalman filter and nonparametric methods such as Particle Filters, seem feasible as brain source localization tools, [2], [3]. However, these techniques are still at a very initial explorative stage and further investigations are required. While both deterministic and statistical parametric approaches have their advantages, hybrid solutions seem to provide reasonable compromise between complexity and computational resources. The hybrid beamformer-particle filter approach for estimation of brain sources based on EEG measurements is the focus of this paper and recent work by the same authors [4], [5].

The hybrid method we propose is inspired by the work of Mohseni et al., [6]. However, in [6] a single-core Beamformer (BF) is used. The main limitation of the single-core BF is that it assumes the estimated dipoles are not correlated. This assumption is not true in general. In Brookes et. al. [7] and Diwakar et al. [8], a dual-core Beamformer is proposed to consider two simultaneously activated sources into a single spatial filter. Inspired by the methodology of Diwakar, we propose an adaptive Beamformer with multiple constraints (Multi-core Beamformer) by adding null-constraints in the potentially correlated source locations. The proposed algorithm simultaneously solves for the inner brain source locations and their waveforms. This is in contrast to techniques which first find the source positions and then estimate the source signals [9].

This paper is organized as follows: In Section 2, the PF framework is outlined. Section 3 formulates the EEG state-space model in order to apply the particle filter, based on physiological specifications. The Beamforming as a spatial filter is introduced in section 3. The proposed method, Multi-core Beamformer Particle Filter (Multi-core BPF), for recursive estimation of the source locations and waveforms is laid out in Section 5. In Section 6 the Multi-core BPF is applied to simulated and real EEG data and compared with alternative solutions. Section 7 concludes the paper.

2. The Particle Filter

Many problems in statistical signal processing, time-series analysis and control can be stated in a state-space form. A system transition function describes the prior distribution of a hidden Markov process according to the model:

$$\boldsymbol{x}_{k+1} = f_k\left(\boldsymbol{x}_k, \boldsymbol{w}_k\right). \tag{1}$$

Here, f_k is the system transition function and $\boldsymbol{w}_k$ is a zero-mean, white noise sequence of known pdf, independent of past and current states. Measurements $\boldsymbol{z}_k$ are available at discrete times *k*, relating to the state vector $\boldsymbol{x}_k$ via the observation equation:

$$\boldsymbol{z}_k = h_k\left(\boldsymbol{x}_k, \boldsymbol{v}_k\right), \tag{2}$$

where h_k is the measurement function and $\boldsymbol{v}_k$ is another zero-mean, white noise sequence of known pdf, independent of past and present states and the system noise.

Within a Bayesian framework, all relevant information about the state vector, given observations up to time *k*, can be obtained from the posterior distribution $p\left(\boldsymbol{x}_k \middle| \boldsymbol{z}_{1:k}\right)$, where $\boldsymbol{z}_{1:k} = \left\{\boldsymbol{z}_1, \cdots, \boldsymbol{z}_k\right\}$. This distribution may be obtained recursively in two steps: prediction and update. Suppose that the posterior distribution at the previous time index $k-1$, $p\left(\boldsymbol{x}_{k-1} \middle| \boldsymbol{z}_{1:k-1}\right)$, is available. Then, using the system transition model, we can obtain the prior pdf of the state at time *k* as follows:

$$p\left(\boldsymbol{x}_k \middle| \boldsymbol{z}_{1:k-1}\right) = \int p\left(\boldsymbol{x}_k \middle| \boldsymbol{x}_{k-1}\right) p\left(\boldsymbol{x}_{k-1} \middle| \boldsymbol{z}_{1:k-1}\right) d\boldsymbol{x}_{k-1}. \tag{3}$$

When a measurement $\boldsymbol{z}_k$, at time step *k*, is available, the prior is updated via Bayes rule:

$$p\left(\boldsymbol{x}_k \middle| \boldsymbol{z}_{1:k}\right) = \frac{p\left(\boldsymbol{z}_k \middle| \boldsymbol{x}_k\right) p\left(\boldsymbol{x}_k \middle| \boldsymbol{z}_{1:k-1}\right)}{p\left(\boldsymbol{z}_k \middle| \boldsymbol{z}_{1:k-1}\right)}, \tag{4}$$

where the denominator is a normalizing factor and the conditional pdf of $\boldsymbol{z}_k$ given $\boldsymbol{x}_k$ is defined by the measurement model in (2).

The recurrence equations in (3) and (4) constitute the solution to the Bayesian recursive estimation problem. If the functions f_k and h_k are linear and the noises $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$ are Gaussian with known variances, then an analytic solution to the Bayesian recursive estimation problem is given by the well-known Kalman filter. In the EEG source localization problem, however, the measurement function h_k is non-linear, or, in other words, the EEG measurements $\boldsymbol{z}_k$ are non-linear functions of the source locations $\boldsymbol{x}_k$.

In order to deal with the non-linear and/or non-Gaussian realities, two main approaches have been adopted: parametric and non-parametric. The parametric techniques are based on extensions of the Kalman filter by linearizing non-linear functions around the predicted values. The non-parametric techniques are based on sequential Monte Carlo methods and particularly the particle filter (PF). Unlike the Kalman filter, which propagates the mean and covariance of the Gaussian posterior density, the PF uses a set of random samples, called particles, to estimate the posterior distribution of the state. Specifically, the posterior is approximated by a set of weighted particles (hence the name particle filter) as:

$$p\left(\boldsymbol{x}_k \middle| \boldsymbol{z}_{1:k}\right) \approx \sum_{i=1}^{N} \pi_k^{(l)} \delta\left(\boldsymbol{x}_k - \boldsymbol{x}_k^{(l)}\right), \tag{5}$$

Here, N is the total number of particles, $\pi_k^{(l)} = w_k^{(l)} / \sum_{l=1}^{N} w_k^{(l)}$ is the normalized weight for particle *l* at time *k*. Ideally, the particles are required to be sampled from the true distribution $p\left(\boldsymbol{x}_k \middle| \boldsymbol{z}_{1:k}\right)$, which is not available. Therefore, another distribution, referred to as the importance distribution, or the proposal distribution $q(\mathrm{x}_n \mid \mathrm{x}_{n-1}, \mathrm{y}_n)$, is used. Theoretically, the only condition on the importance distribution is that its support includes the support of the posterior distribution. In practice, the number of particles is finite and the importance distribution should be chosen to approximate the posterior distribution. The importance weights are given by:

$$w_k^{(l)} = w_{k-1}^{(l)} \frac{p\left(\boldsymbol{z}_k \middle| \boldsymbol{x}_k^{(l)}\right) p\left(\boldsymbol{x}_k^{(l)} \middle| \boldsymbol{x}_{k-1}^{(l)}\right)}{q\left(\boldsymbol{x}_k^{(l)} \middle| \boldsymbol{x}_{k-1}^{(l)}, \boldsymbol{z}_{1:k}\right)}, \tag{6}$$

For instance, if the importance distribution is given by the prior density, $q\left(\boldsymbol{x}_k^{(l)} \middle| \boldsymbol{x}_{k-1}^{(l)}, \boldsymbol{z}_{1:k}\right) = p\left(\boldsymbol{x}_k^{(l)} \middle| \boldsymbol{x}_{k-1}^{(l)}\right)$, then Eq. (6) reduces to:

$$w_k^{(l)} = w_{k-1}^{(l)} p\left(\boldsymbol{z}_k \middle| \boldsymbol{x}_k^{(l)}\right). \tag{7}$$

Given a discrete approximation to the posterior distribution, one can then proceed to a filtered point estimate such as the mean of the state at time *k*:

$$\hat{\boldsymbol{x}}_k = \sum_{l=1}^{N} \pi_k^{(l)} \boldsymbol{x}_k^{(l)}. \quad (8)$$

The main advantage of the particle filter is that no restrictions are placed on the functions f_k and h_k, or on the distribution of the system and measurement noise. Moreover, the algorithm is quite simple and easy to implement. Notably, it can be implemented on massively parallel computers, raising the possibility of real time operation with very large sample sets.

3. The EEG State-Space Model

In order to apply the particle filtering, the state-space model of the EEG source localization problem has to be defined. Assuming that the brain electrical activity has been originated by a number of active dipoles, the EEG signal $\boldsymbol{z}_k$ from n_z sensors at time *k* (the forward EEG model) can be expressed by [1]:

$$\boldsymbol{z}_k = \boldsymbol{L}(\boldsymbol{x}_k)\boldsymbol{s}_k + \boldsymbol{v}_k, \quad (9)$$

where $\boldsymbol{x}_k$, is the state vector that represents the dipole geometrical positions at time *k*. For example, for two dipoles, the state vector is

$$\boldsymbol{x}_k = \left[x_{1k}, y_{1k}, z_{1k}, x_{2k}, y_{2k}, z_{2k}\right]^t, \quad (10)$$

L is the lead field dipole matrix at time *k*, $\boldsymbol{s}_k$, is the vector of the 3D source signal and $\boldsymbol{v}_k$ is a white Gaussian noise with variance σ_v^2. From Equation (9), the likelihood of each measurement can be obtained:

$$\mathcal{L}\left(\boldsymbol{z}_k \middle| (\boldsymbol{x}_k, \boldsymbol{s}_k)\right) \propto \exp\left[-\frac{\left(\boldsymbol{z}_k - \boldsymbol{L}(\boldsymbol{x}_k)\boldsymbol{s}_k\right)^t \boldsymbol{R}_{z_k}^{-1}\left(\boldsymbol{z}_k - \boldsymbol{L}(\boldsymbol{x}_k)\boldsymbol{s}_k\right)}{2}\right], \quad (11)$$

where $\boldsymbol{R}_{z_k}$ is the covariance matrix of the measurement vector $\boldsymbol{z}_k$, $\propto$ denotes "proportional to". The goal is to estimate $\boldsymbol{x}_k$, given $\boldsymbol{z}_k$. Assuming a lack of *a priori* knowledge of the dipole location, the state transition is assumed to be a random walk in the source localization space:

$$\boldsymbol{x}_k = \boldsymbol{x}_{k-1} + \boldsymbol{w}_k, \quad (12)$$

where $\boldsymbol{w}_k$ is a Gaussian white noise with variance σ_w^2. Thus, the complete state-space model of the dipole source localization is the following:

$$\begin{cases} \boldsymbol{x}_k = \boldsymbol{x}_{k-1} + \boldsymbol{w}_k & \text{state transition model} \\ \boldsymbol{z}_k = \boldsymbol{L}(\boldsymbol{x}_k)\boldsymbol{s}_k + \boldsymbol{v}_k & \text{observation model.} \end{cases} \quad (13)$$

In model (13), the source waveforms $\boldsymbol{s}_k$ will be estimated by the beamforming method discussed in the next section.

4. Multi-core Beamforming for Correlated Source Localisation

Beamforming (BF) approach was successfully applied in a variety of neuroimaging studies [10], [11]. In the present work, we want to estimate the dipole waveforms $\boldsymbol{s}_k$ using only EEG measurements. The idea is to build a spatial filter that would pass signals from the location of interest with a unit gain, while nulling signals from elsewhere (*i.e.*, it is insensitive to activity from other brain regions). The BF filter consists of weight coefficients that, when multiplied by the electrode measurements, give an estimate of the dipole moment at time *k*, *i.e.*,

$$\boldsymbol{s}_k = W^t z_k, \quad (14)$$

where W^t is the weighting matrix.
Among a number of criteria for choosing the optimum weights, the linearly constrained minimum variance (LCMV) BF provides an adaptive alternative in which the spatial filter is optimized with respect to the measured data, [10]. The objective is to optimize the BF response with respect to a prescribed criterion, so that the output $\boldsymbol{s}_k$ contains minimal contribution from noise and interference. The nulling effect is achieved by minimizing the variance of the filter output subject to a unit gain constraint at the desired location. The constrained optimization problem can be expressed as:

$$\begin{aligned} &\boldsymbol{W}^* = \underset{\boldsymbol{W}}{\operatorname{argmin}}\ Tr\left[\boldsymbol{W}^t \boldsymbol{R}_{z_k} \boldsymbol{W}\right] \\ &\text{subject to } \boldsymbol{W}^t \boldsymbol{L}(\boldsymbol{x}_k) = \boldsymbol{I}. \end{aligned} \quad (15)$$

The optimal solution can be derived by constrained minimization using Lagrange multipliers,

$$\boldsymbol{W}^* = \boldsymbol{R}_{z_k}^{-1}\boldsymbol{L}(\boldsymbol{x}_k)\left[\boldsymbol{L}(\boldsymbol{x}_k)^t \boldsymbol{R}_{z_k}^{-1}\boldsymbol{L}(\boldsymbol{x}_k)\right]^{-1}. \quad (16)$$

The conventional (single-core) Beamformer approach, described above, has an important limitation when spatially distinct yet temporally correlated sources are present in the EEG signal, [10]. The main assumption of the beamformer method is that the activity at the target location is not linearly correlated with activity at any other location. However, several studies of functional connectivity have suggested that temporal correlation relates to the communications among cortical areas. For example, such high correlations occur during evoked sensory responses in which the sensory information is transmitted to both left and right auditory cortices simultaneously, which result in almost perfectly correlated activities in the two hemispheres. Correlated activities can also be observed in symmetric regions of the left and right hemispheres of the motor cortex.

Different modifications of the single-core BF attempt to compensate for this limitation. The temporal correlation $M_{i,j}(f)$ of a pair of (i,j) dipoles is quantified by the magnitude-squared cross spectrum $S_{i,j}(f)$ divided by the power spectra of both dipole moments $S_{i,i}(f)$ and $S_{j,j}(f)$:

$$M_{i,j}(f) = \frac{\left|S_{i,j}(f)\right|^2}{S_{i,i}(f)S_{j,j}(f)}. \quad (17)$$

The correlation is bounded between 0 and 1, where $M_{i,j}(f) = 1$ indicates a perfect linear relation between dipoles d_i and d_j at frequency *f*.

Dynamic imaging of coherent sources (DICS) is proposed in [12] where the spatial filter weighting matrix explicitly takes into account the estimated correlation quantified by Eq. (17). The authors conclude that high coherence results in a large error in the estimation of the dipole location. Low signal to noise ratio (SNR) additionally deteriorates the estimation of spatially close and temporally correlated dipoles. Correlated dipoles can be reliably localized if the distance between them is sufficiently high. DICS computes the cross spectral densities for any given location (from a dense grid of points) and all pair combinations of grid dipoles.

An alternative solution for dealing with the problem of correlated source activities is to include the other source location in the forward model to obtain a bilateral beamfomer. This idea has been further developed into dual-core beamformers allowing the correlated sources in any location. For example, Brookes *et. al.* [7] proposed a spatial filtering technique by linearly combining source lead fields. Recently, Diwakar *et al.* [8] have further developed this idea by proposing a new dual-core beamformer that incorporates the leadfield vectors of two simultaneously activated sources into a single spatial filter.

Inspired by the methodology of Diwakar, we adopted an adaptive beamformer based on the LCMV algorithm with multiple constraints (multi-core beamformer) by adding null-constraints in the potentially correlated source locations. The optimization problem is solved using the method of Lagrange multipliers with multiple constraints:

$$\begin{aligned} &\min_{\boldsymbol{W}} \; Tr\left[\boldsymbol{W}^t \boldsymbol{R}_x \boldsymbol{W}\right] \\ &\text{subject to } \; \boldsymbol{W}^t \boldsymbol{L}\left(\boldsymbol{d}_1\right) = \boldsymbol{I} \\ &\qquad\qquad\quad \boldsymbol{W}^t \boldsymbol{L}\left(\boldsymbol{d}_2\right) = \boldsymbol{0} \\ &\qquad\qquad\qquad\quad \vdots \\ &\qquad\qquad\quad \boldsymbol{W}^t \boldsymbol{L}\left(\boldsymbol{d}_n\right) = \boldsymbol{0}. \end{aligned} \quad (18)$$

The conventional BF is characterized with high computational costs due to the scanning solution over a three dimensional source grid with thousands of nodes (potential source locations). The BF modifications to account for correlated sources increase even more the computational burden because of the additional cross correlation estimation for all pair combinations of grid dipoles. Instead, the combined solution proposed in this work (multi-core BF and PF) has the advantage of considering temporal source correlation in the framework of relatively small number of particle-like sources, but whose level of correlation is unknown a priori. The price to pay for using multiple constraints is that the number of degrees of freedom decreases and the beamformer becomes less adaptive to other unknown sources.

5. The Multi-core Beamformer-Based Particle Filter

The Multi-core Beamformer Particle Filter (Multi-core BPF) is a hybrid (statistical-deterministic) framework for reconstruction of correlated source. This is a recursive procedure that first estimates the locations of temporally correlated brain sources (using particle filter) and then estimates their corresponding waveforms (using multi-core beamforming spatial filter). The algorithm is summarized below.

Multi-core Beamformer Particle Filter for temporally correlated source localization

1) *Initialization*

   a) $k = 0$, for $l = 1,\ldots,N$, where *N* denotes the total number of particles.

   Generate samples $\boldsymbol{x}_0^{(l)} \sim p\left(\boldsymbol{x}_0\right)$ and set initial weights $\pi_0^{(l)} = 1/N$.

   b) for $k = 1,2,\ldots,$

2) *Prediction step*

   for $l = 1,\ldots,N$, generate samples according to the state transition model in Eq. (12):

   $\boldsymbol{x}_k^{(l)} = \boldsymbol{x}_{k-1}^{(l)} + \boldsymbol{w}_k^{(l)}$ where $\boldsymbol{w}_k^{(l)} \sim \boldsymbol{\mathcal{N}}\left(\boldsymbol{0}, \sigma_w^2 \boldsymbol{I}\right)$

3) *Multi-core Beamforming*

   a) Compute the lead field matrix $\boldsymbol{L}\left(\boldsymbol{x}_k^{(l)}\right)$ for each predicted dipole at step 2 by solving the Maxwell equations [1].

   b) Find the optimal spatial filter weights using Eq. (18). Consider the location of each estimated dipole d_i $\left(i = 1,\ldots,M\right)$ as the targeted direction and the other $M-1$ dipoles as correlated with d_i to compute the weighted vector associated with it.

c) Compute the source waveforms $\boldsymbol{s}_k^{(l)}$ according to Eq. (14).

4) *Measurement update*

Evaluate the particle weights:

a) for $l=1,\ldots,N,$ on the receipt of a new measurement, compute the weights

$$w_k^{(l)} = w_{k-1}^{(l)} \mathcal{L}\left(\boldsymbol{z}_k \left| \left(\boldsymbol{x}_k^{(l)}, \boldsymbol{L}\left(\boldsymbol{x}_k^{(l)}\right), s_k^{(l)} \right) \right. \right). \tag{19}$$

The likelihood $\mathcal{L}\left(\boldsymbol{z}_k \left| \left(\boldsymbol{x}_k^{(l)}, \boldsymbol{L}\left(\boldsymbol{x}_k^{(l)}\right), s_k^{(l)} \right) \right. \right)$ is calculated using Eq. (11).

b) for $l=1,\ldots,N,$ normalize the weights,

$$\pi_k^{(l)} = w_k^{(l)} / \sum\nolimits_{l=1}^{N} w_k^{(l)}. \tag{20}$$

5) Evaluate the posterior mean $E\left[\boldsymbol{x}_k \left| \boldsymbol{z}_{1:k} \right. \right]$ as the estimate of the state at iteration k

$$\hat{\boldsymbol{x}}_k = E\left[\boldsymbol{x}_k \left| \boldsymbol{z}_{1:k} \right. \right] = \sum_{l=1}^{N} \pi_k^{(l)} \boldsymbol{x}_k^{(l)}. \tag{21}$$

6) Compute the effective sample size:

$$N_{eff} = 1 / \sum\nolimits_{l=1}^{N} \left(\pi_k^{(l)}\right)^2.$$

7) Selection step (resampling):

if $N_{eff} < N_{thresh}$: multiply/suppress samples $\left\{\boldsymbol{x}_k^{(l)}\right\}$ with high/low weights $\pi_k^{(l)}$, in order to obtain N new random samples approximately distributed according to the posterior state distribution.

8) Stopping criteria:

$\left|\hat{\boldsymbol{x}}_k - \hat{\boldsymbol{x}}_{k-1}\right| < \varepsilon,$ where ε is a fixed threshold

6. Simulation Results

The proposed approach is assessed by simulation experiments assuming the EEG signals are generated by a limited number of focal sources. Three-shell spherical head model was created based on the following assumptions:

The head model consists of three concentric spherical shells with the enclosed space among them representing the scalp, skull and brain. The model dimensions are scaled to a realistic human head with an outer shell radius of 10 cm, scalp radius of 9.2 cm and skull radius of 8.7 cm.

Each layer is considered as homogeneous and isotropic, *i.e.*, the conductivity is constant and with no preferred direction. The conductivity values used for the head model were selected from studies on electrical impedance tomography aiming to create an electrical conductivity map of a volume: scalp 0.33 S/m, skull 0.0165 S/m and brain 0.33 S/m.

The distribution of the electrodes on the scalp follows the standard 10/20 International system with an array of 30-electrodes: Fp1, AF3, F7, F3, FC1, FC5, C3, CP1, CP5, P7, P3, Pz, PO3, O1, Oz, O2, PO4, P4, P8, CP6, CP2, C4, FC6, FC2, F4, F8, AF4, Fp2, Fz, Cz.

The coordinates are defined with respect to a reference frame whose origin is located at the centre of the sphere: the *x*-axis pointing in the direction of the right-ear, the *y*-axis pointing in the front of the head and the *z*-axis is taken to be vertical.

White noise was added to the generated EEG signals representing the effect of external sources not generated by brain activity, but by some disturbance (*e.g.*, movements of muscles). The noise power was defined for different signal-to-noise ratios (SNR). The SNR is defined in the sensor domain as the total power of the signal divided by the total power of the noise added to the signal. The total searchable source space is simulated with a fixed and uniform dipole-grid with 5-mm spacing in each direction. The leadfield matrix is computed off-line for each grid dipole. A grid of 21012 dipoles is used in the simulations.

6.1 Dipole localisation results

Sinusoidal waveforms with amplitudes 0.1 and frequencies 10 Hz and 15 Hz are assumed to be the brain signals originating from the two dipoles (d_1 and d_2). For the initial state vector, $N=500$ samples are randomly generated from a normal distribution in the interval $\boldsymbol{x}_0 \in \left[\min(D), \max(D)\right]$ with $D=\left\{d_j = \left[x_j, y_j, z_j\right]\right\}$.

The particle filter finds the brain source coordinates $\boldsymbol{x} = \left[x_1, y_1, z_1, x_2, y_2, z_2\right]$ as described in Section 5. In the simulations, the sources are randomly generated and, therefore, they may or may not coincide with the dipole grid that describes the head model. We consider three cases: (i) the two brain sources are located on the dipole grid, (ii) only one brain source coincides with a dipole grid, and (iii) none of the brain sources is located on the dipole grid. Figures 1, 2 and 3 show the absolute estimation error for the three cases with all simulations running for 200 iterations.

Observe that the absolute estimation error with respect to each space coordinate (x, y, z) converges to zero after a few iterations if the original brain sources are located on the dipole grid of the head model. Otherwise, the estimation of the localization of non-grid-dipoles may end with a small steady-state error. The proposed algorithm guarantees a zero estimation error when the head grid model becomes infinitely fine.

6.2 Multi-core BPF versus single-core BPF and full PF

In order to validate the Multi-core BPF, we compare it with the two alternative techniques, single-core BPF and the full

PF, from which the proposed method originated. The experiments were performed with the following control conditions: the neural activity from a-pair of correlated dipole sources with 95% ($M = 0.95$) and 30% correlation ($M = 0.3$) were simulated as sinusoidal base waves with amplitudes 0.1 and frequencies 3Hz and 5Hz over 0.5 sec. The performance was evaluated at low SNRs (3dB and 8dB). The target dipoles (ground truth) were taken from the predefined grid at the following (x, y, z) coordinates: $d_1 : (0.0116, 0.0767, 0.019)m$ and $d_2 : (-0.0116, -0.0767, 0.0095)m$ with a dominant direction of propagation along the *x*-axis for d_1 and along the *y*-axis for d_2 defined by the following vectors: $dir_1 : (0.8, 0.1, 0.1)$ and $dir_2 : (0.1, 0.8, 0.1)$.

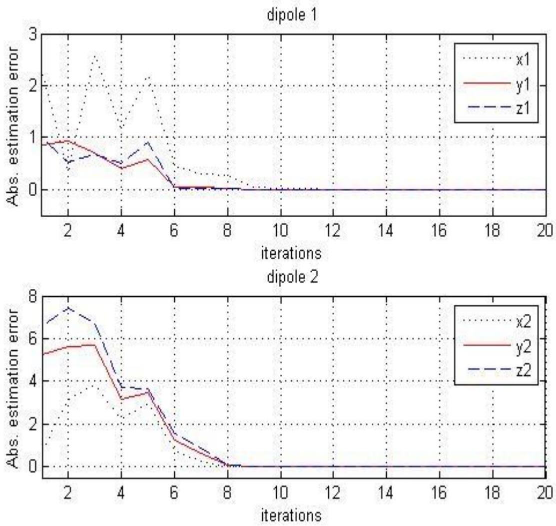

Figure 1: Absolute estimation error of the dipole locations when the two brain sources are located on the dipole grid.

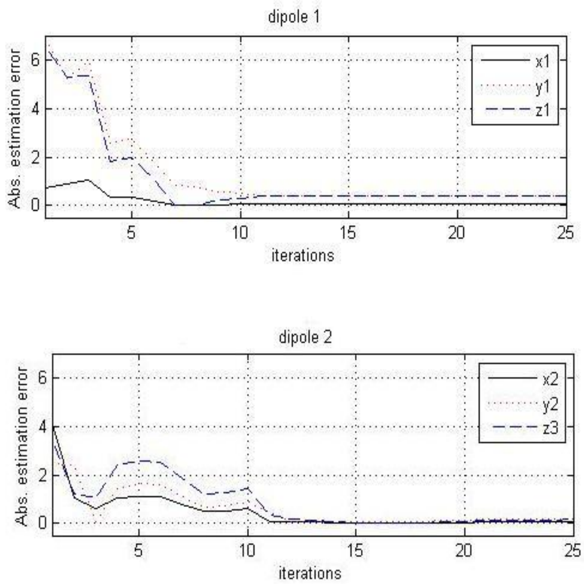

Figure 2: Absolute estimation error of the dipole locations when only the second brain source coincides with a dipole grid.

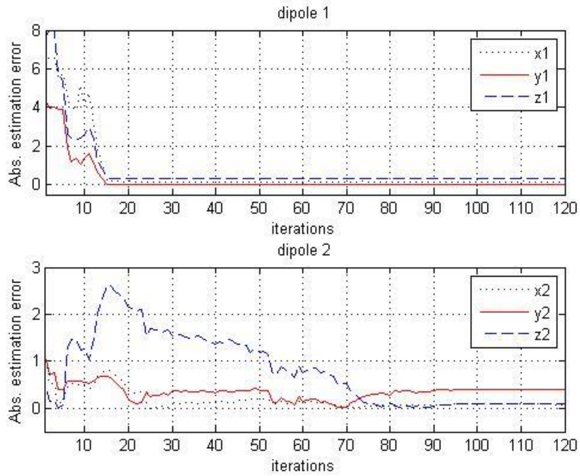

Figure 3: Absolute estimation error of the dipole locations when none of the brain sources is located on the dipole grid.

First, the effect of the dipole correlation (expressed by *M*) on the beamformer was evaluated (see Figs. 4 and 5). Note that the simulation of dipole correlation changes the sine shape of the base signal. The single-core BF and the multi-core BF provide very similar estimations for uncorrelated dipoles. The higher the correlation level ($M = 0.95$), the more biased are the estimations of the single-core BF as can be seen in Figure 6. This is due to the filter weight matrix that was computed assuming the source time-courses come from uncorrelated generators.

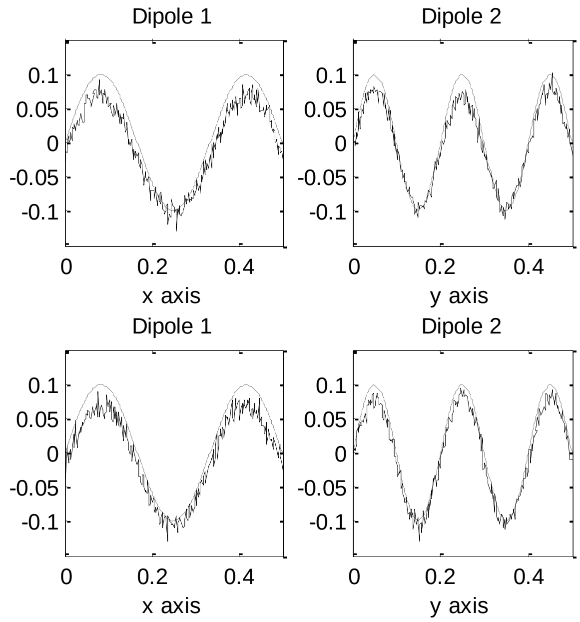

Figure 4: Source waveform estimation by beamforming for uncorrelated dipoles: the original (dotted line) and the

estimated curve (bold line) for dipole 1 (left) and dipole 2 (right) using the Single-core BF (top plots) and the Multi-core BF (bottom plots) with $\mathrm{SNR} = 3dB$.

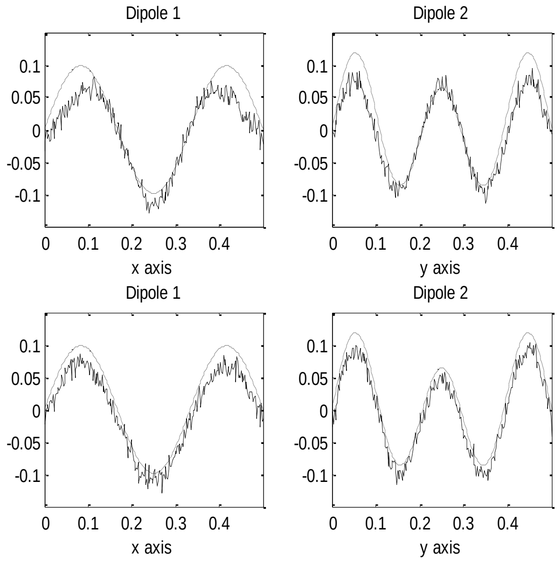

Figure 5: Source waveform estimation by beamforming for $M = 0.3$ (low correlation): the original (dotted line) and the estimated curve (bold line) for dipole 1 (left) and dipole 2 (right) using the Single-core BF (top plots) and the Multi-core BF (bottom plots) with $\mathrm{SNR} = 3dB$.

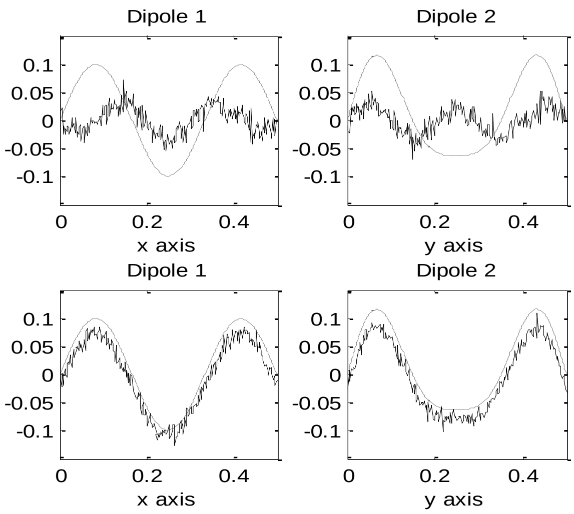

Figure 6: Source waveform estimation by beamforming for $M = 0.95$ (high correlation): the original (dotted line) and the estimated curve (bold line) for dipole 1 (left) and dipole 2 (right) using the Single-core BF (top plots) and the Multi-core BF (bottom plots) with $\mathrm{SNR} = 3dB$.

Table 1 summarizes the spatial mean squared localization errors (MSE) under varying SNR and varying correlation levels *M*. The MSE is obtained from the space distance $\left(\sqrt{(\hat{x}-x)^2+(\hat{y}-y)^2+(\hat{z}-z)^2}\right)$ in millimetres between the true (x, y, z) and the estimated sources $(\hat{x}, \hat{y}, \hat{z})$ across 10 Monte Carlo simulations. Even from very noisy EEG data $(\mathrm{SNR} = 3dB)$, without any prior assumption for the true location of the dipoles, the Multi-core BPF provides estimation within 3-5 mm error distance. The single-core BPF can achieve competitive accuracy, but only for dipoles with low or no temporal correlation. The full PF is less sensitive to dipole correlation and noise. The PF estimation error is relatively high. However, if the number of the particles is higher (only 500 in the present scenario), it has the potential to recover better the dipole location. The price to be paid is the significant amount of memory and computational power particularly when the number of estimated dipoles increases.
Figure 7 visualizes the normalized weights computed over the recursive PF estimation for some of the iterations *k*. Note that, based on the current likelihood value at each iteration, only few of the particles (from $N = 500$ particles in total) are pointed out as the most probable candidates for the location of the dipoles. This reduces significantly the computational efforts associated with the exhaustive search over the complete dipole grid conducted by the full beamforming approach or other deterministic parametric methods for brain source localization.

Method	$\mathrm{SNR} = 3dB$			
	Dipole 1		Dipole 2	
	$M = 0.95$	$M = 0.3$	$M = 0.95$	$M = 0.3$
Full PF	8.2	8.3	7.3	7.6
Single-core BPF	12.2	3.95	9.97	3.3
Multi-core BPF	3.4	5.42	1.8	4.41
Method	$\mathrm{SNR} = 8dB$			
	Dipole 1		Dipole 2	
	$M = 0.95$	$M = 0.3$	$M = 0.95$	$M = 0.3$
Full PF	6.9	6.7	5.8	5.3
Single-core BPF	11.5	3.3	8.7	3.1
Multi-core BPF	2.8	4.1	1.5	3.6

Table 1: Spatial mean squared localization errors (MSE) in millimetres under varying SNR and correlation levels M.

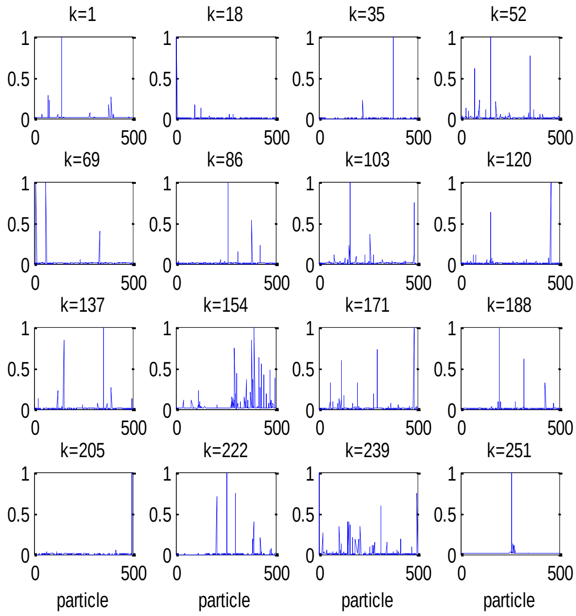

Figure 7: Normalized weights (Eq. 20) computed over the recursive PF estimation.

7. Conclusions

This paper proposes a general framework, termed multi-core Beamformer Particle Filter (multi-core BPF), for solving the ill-posed EEG inverse problem. The method combines a particle filter (statistical approach) for estimation of the spatial location and a multi-core beamformer (deterministic approach) for estimation of temporally correlated dipole moments in a recursive framework. The intuition behind it is to benefit from the advantages of both deterministic and statistical inverse problem solvers in order to improve the estimation accuracy without increasing the complexity and computational cost.

Our simulations show that the proposed algorithm can reconstruct reliably the few most active (the dominant) brain sources that have generated the registered EEG measurements. The main advantage of the method is that it explicitly takes into consideration potential temporal correlation between the dipoles.

Acknowledgments

This work was partially funded by FEDER through the Operational Program Competitiveness Factors - COMPETE and by National Funds through FCT - Foundation for Science and Technology, in the context of the projects FCOMP-01-0124-FEDER-022682 (FCT reference PEst-C/EEI/UI0127/2011) and Incentivo/EEI/UI0127/2013. The authors acknowledge also the support of the UK Engineering and Physical Sciences Research Council (EPSRC) via grant Bayesian Tracking and Reasoning over Time (BTaRoT) EP/K021516/1.

References

1. S. Sanei, J. Chambers, EEG Signal Processing, John Wiley & Sons, 2007.
2. J. Antelis, J.Minguez, Dynamic solution to the EEG source localization problem using Kalman filters and particle filters, in: Proceedings of the International Conference of the IEEE Engineering in Medicine and Biology Society, pp. 77-80, 2009.
3. H. R. Mohseni, E. L. Wilding, S. Sanei, Sequential Monte Carlo techniques for EEG dipole placing and tracking, in: Proceedings of the Sensor Array and Multichannel Signal Processing Workshop, pp. 95-98, 2008.
4. P. Georgieva, N. Bouaynaya, L. Mihaylova, F. Silva (2013). Bayesian approach for reconstruction of moving brain dipoles. Int. Conf. on Image Analysis and Recognition (ICIAR), June 26-28, 2013, Póvoa de Varzim, Portugal, vol. "Lecture Notes in Computer Science" (LNCS) series of Springer Verlag , pp.565-572.
5. P. Georgieva, F. Silva, M. Milanova and N. Kasabov (2013), Chapter "EEG Signal Processing for Brain Computer Interfaces", N. Kasabov, (Ed.) Springer Handbook of Bio- and Neuroinformatics" (HBBNI), ISBN 978-3-642-30573-3.
6. H.R. Mohseni, F. Ghaderi, E.L. Wilding, S. Sanei, A beamforming particle filter for EEG dipole source localization, in: Proceedings of the IEEE International Conference on Acoustics, Speech and Signal Processing, pp. 337-340, 2009.
7. M. Brookes, C. Stevenson, G. Barnes, A. Hillebrand, M. Simpson, S. Francis, P. Morris, Beamformer reconstruction of correlated sources using a modified source model, Neuroimage 34 (2007) 1454-1465.
8. M. Diwakar, M.-X. Huang, R. Srinivasan, D. Harrington, A. Robb, A. Angeles, L. Muzzatti, R. Pakdaman, T. Song, R. Theilmann, R. Lee, Dual-core beamformer for obtaining highly correlated neural networks in MEG, Neuroimage 54 (2011) 253-263.
9. V. Litvak, K. Friston, Electromagnetic source reconstruction for group studies, Neuroimage, 42 (4) (2008) 1490-1498.
10. B. V. Veen, W. V. Drongelen, M. Yuchtman, A. Suzuki, Localization of brain electrical activity via linearly constrained minimum variance spatial filter, IEEE Trans. Biomed. Eng. 44 (9) (1997) 867-880.
11. M. Huang, J. Shih, R. Lee, D. Harrington, R. Thoma, M. Weisend, F. Hanion, K. Paulson, T. Li, K. Martin, G. Miller, J. Canive, Commonalities and differences among vectorized beamformers in electromagnetic source imaging, Brain Topogr. 16 (3) (2004) 139-158.
12. J. Gross, J. Kujala, M. Hamalainen, L. Timmermann, A. Schnitzler, R. Salmelin, Dynamic imaging of coherent sources: studying neural interactions in the human brain, National Academy of Sciences, 98 (2001) 694-699.